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THE EFFECT OF TEMPERATURE ON TRACTION IN ELASTOHYDRODYNAMIC LUBRICATION

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An experimental study has been made of the effect of temperature on traction in elastohydrodynamic conditions with a range of nine fluids. The pressure coefficient of viscosity falls with increase of temperature and this, combined with the fall in the viscosity at atmospheric pressure, leads to very great changes in the effective viscosity within the elastohydrodynamic contact. Thus whereas at low rates of shear and low temperature the fluid in the high pressure zone may behave as an elastic solid, when the temperature is raised it reverts to its liquid form with a viscosity of the order of 10^2 Pa s or less. At high rates of shear, the fluid behaves as a non-Newtonian liquid of the Eyring type at all temperatures within the range of the experiments, i.e. from 30 to 110 °C.

The viscosity, η, of a fluid of the Eyring type can be expressed as

$$\eta = \frac{\eta_0 \exp(\alpha p)}{\sinh(\tau/\tau_0)} \frac{\tau}{\tau_0}, \tag{I}$$

where η_0 is the viscosity at atmospheric pressure, α is the pressure coefficient, p is the pressure, τ is the shear stress and τ_0 is a characteristic shear stress determining the non-Newtonian behaviour. At high rates of shear, and high pressures, $\sinh(\tau/\tau_0)$ approximates to the exponential form $\exp(\tau/\tau_0)$ and the value of the traction coefficient (T/W) is then given by the expression

$$\frac{T}{W} = \bar{\alpha}\tau_0 - \frac{\tau_0}{p} \ln \frac{\tau_0}{2\eta_0 \dot{s}}, \tag{II}$$

where $\dot{s}$ is the rate of shear and $\bar{\alpha}$ is the mean value of α over the range of pressure. It has been found that $\bar{\alpha}\tau_0$ is sensibly independent of temperature which suggests that the limiting coefficient of traction at sufficiently high pressure should also be independent of temperature. Since $\bar{\alpha}$ falls and τ_0 rises with temperature, the effect of an increase in temperature is to raise the pressure at which a given value of the traction coefficient is achieved.

In some fluids the molecules appear to associate into clusters. The clusters dissociate when the temperature is raised and for these fluids τ_0 rises nearly exponentially with temperature. For two of the fluids the unit of flow is the individual molecule. τ_0 is then directly proportional to the absolute temperature in agreement with the Eyring theory.

At low pressures and high temperatures, the approximate relation of equation (II) ceases to be applicable. The full expression of equation (I) has then to be used and the variation of pressure over the region of contact has to be taken into account.

1. Introduction

Elastohydrodynamic lubrication is the mechanism of lubrication of gears and rollers. The traction between rollers is of particular relevance to the operation of continuously variable transmission systems and it depends upon shear properties of the lubricant that cannot be seen in less demanding circumstances. Within the zone of contact the lubricant is subjected to very severe shear stresses and rates of shear and it is now generally accepted that in such conditions the oil ceases to behave as a Newtonian liquid. Experimental evidence shows that the non-Newtonian behaviour takes more than one form. Thus Johnson & Roberts (1974) have shown that mineral oils undergo a transition from viscous to elastic behaviour over the pressure range 0.5–0.75 GPa. Similar behaviour has since been observed for a much wider range of fluids (Hirst & Moore 1979) but at transition pressures that vary with the pressure coefficient of viscosity. This is an example of viscoelastic behaviour. The relaxation time, λ, of the lubricant is equal to the ratio, η/G, of its viscosity to its shear modulus and since the viscosity rises approximately exponentially with pressure and the modulus only linearly, λ also increases very rapidly. Eventually it exceeds the time taken by the lubricant to pass through the zone of contact. At 30 °C, departures from viscous behaviour first become evident when the viscosity reaches a value of about 10^4 Pa s (Johnson & Cameron 1967/68) and at a viscosity of about 10^5 Pa s the transition to elastic behaviour appears to be complete (Hirst & Moore 1979). This is the first main form of non-Newtonian behaviour.

The second form arises from the magnitude of the shear stresses. It was shown by the authors (Hirst & Moore 1974) that when a steadily increasing shear stress is applied to a lubricant there comes a point at which the relation between shear stress and shear rate ceases to be linear; thereafter the shear rate increases more rapidly than the stress. The authors showed that this is an intrinsic property of the fluid which can be seen in conditions where shear heating is negligible. The magnitude of the critical stress determining the limit of Newtonian behaviour depends on the fluid and on the pressure but, in general, it lies within the range of stress encountered in typical elastohydrodynamic conditions. In subsequent papers (Hirst & Moore 1975, 1978) the authors showed that nonlinear behaviour is also exhibited in the elastic domain at high pressures and that the relation between stress and strain then resembles the yielding of an elastic solid. At lower pressures the behaviour approximates to that of the Eyring model for a simple liquid (Hirst & Moore 1974).

The complexity of the pattern of the relations between shear stress and shear rate in elastohydrodynamic conditions has long been recognised and many hypotheses have been advanced to account for it. In particular, the fact that at high pressures the elastohydrodynamic film yields rather suddenly at a shear stress of about 10 % of the mean pressure led Smith (1962) to suggest that the film shears as a plastic solid rather than as a viscous liquid. A similar view has been advanced by Plint (1967). However, Johnson & Tevaarwerk (1977) have recently made a theoretical analysis of a model system that incorporated the nonlinear Eyring expression for viscosity into the Maxwell model of a viscoelastic liquid. Their analysis showed that such a description of a liquid predicts each of the characteristic forms of the relation between traction and slip observed in practice. In particular, it predicts a change in form of the relation as pressure is raised, from the gradual departure from linearity expected for viscous liquids to the rapid departure from linearity associated with the yielding of solids.

This rheological model of a nonlinear Maxwell fluid thus offers the hope of unifying a range of apparently diverse phenomena. There remain, however, some difficulties. One is the fact that the values of the elastic modulus derived from elastohydrodynamic experiments are lower than those measured under equilibrium conditions. An obvious difference between the two types of experiment is that the time for which the oil in an elastohydrodynamic contact is under pressure is typically six orders of magnitude smaller than in a static experiment. This had led to the proposal that when pressure is suddenly applied to a lubricant it converts to a glass and so does not assume its equilibrium state (Alsaad *et al.* 1978; Montrose & Heyes 1977). It is important to note, however, that the derivation of elastic moduli from elastohydrodynamic experiments is not straightforward. Allowance must first be made for the contribution to the measured slip due to creep of the rollers. Both this and the assumed distribution of shear stress in the contact greatly affect the magnitude of the derived modulus. A detailed treatment of this problem considerably reduces the disparity between the two types of experiment but does not altogether remove it (Johnson *et al.* 1978).

In the nonlinear region the creep of the rollers becomes unimportant and can be quite adequately corrected for by simple methods (Hirst & Moore 1978). The authors have previously examined a range of widely differing fluids in this régime (Hirst & Moore 1979) and shown that they invariably lose any elastic character and behave simply as non-Newtonian liquids (see also Johnson & Tevaarwerk 1977). Their behaviour can then be described in terms of two parameters, $\bar{\alpha}$, a mean pressure coefficient of viscosity, and τ_0, a shear stress that characterizes the onset of non-Newtonian behaviour.

Previous experiments were all done at or near room temperature. Many engineering systems operate at much higher temperatures and the influence of temperature is thus of some importance. This paper describes an investigation of the effect of temperature on traction in elastohydrodynamic conditions. It is shown that, as the temperature rises, the fall in the maximum traction that a fluid can sustain is surprisingly small considering the change in the magnitude of the Newtonian viscosity. This is because the traction is controlled mainly by the non-Newtonian character of the liquid and while both $\bar{\alpha}$ and τ_0 vary with the temperature, they do so in opposite senses. The limiting traction coefficient, $\bar{\alpha}\tau_0$, is consequently little affected by the rise in temperature. Owing to the rise in τ_0, however, the pressure at which the limiting traction is attained increases.

2. The rheological model

(a) *Nonlinear viscoelasticity*

Eyring's expression for viscosity may be written

$$\eta = \frac{\tau}{2A}\,\frac{\exp\left[(E+pv_p)/k\theta\right]}{\sinh\,(\tau v_\tau/k\theta)}, \tag{1}$$

where A is a frequency term proportional to temperature, E is the activation energy for flow at atmospheric pressure, θ is the absolute temperature and k is Boltzmann's constant; v_p and v_τ are activation volumes for pressure and shear respectively. Writing $k\theta/v_\tau = \tau_0$ and $v_p/k\theta = \alpha$, (1) becomes

$$\eta = \frac{\eta_0 \exp\,(\alpha p)\,(\tau/\tau_0)}{\sinh\,(\tau/\tau_0)}, \tag{2}$$

where η_0 is the Newtonian viscosity at atmospheric pressure. When $\tau \ll \tau_0$ (2) approximates to

$$\eta = \eta_0 \exp\,(\alpha p), \tag{3}$$

which is the Barus expression for viscosity; when $\tau \gg \tau_0$ it approximates to

$$\eta = \frac{2\eta_0 \exp\,(\alpha p)\,(\tau/\tau_0)}{\exp\,(\tau/\tau_0)}. \tag{4}$$

The relaxation time of a viscoelastic fluid, λ, is given by the ratio η/G, where G is the elastic modulus. Thus, when τ is small, the relaxation time is given by

$$\lambda = \eta_0 \exp\,(\alpha p)/G, \tag{5}$$

which shows that λ increases very rapidly with pressure (since G does not). The system therefore becomes elastic at sufficiently high pressures. However, when τ is large ($\tau \gg \tau_0$) the term $\exp\,(\tau/\tau_0)$ in (4) becomes effective and the system reverts to the liquid condition. It is apparent that at high pressures the state of the system is controlled by the magnitudes of the two parameters, $\alpha(=v_p/k\theta)$ and $\tau_0(=k\theta/v_\tau)$. It may also be noted that the product $\alpha\tau_0 = v_p/v_\tau$.

It follows that at low degrees of slip the lubricant may respond to the applied shear either as a Newtonian liquid or as an elastic solid. If the behaviour is viscous, an effective viscosity, $\bar{\eta}$, may be derived from the traction in line contact conditions by means of the relation

$$\bar{\eta} = \frac{T}{2a}\frac{h}{\Delta U}, \tag{6a}$$

where T is the traction per unit disk width, $2a$ is the length of the Hertzian pressure zone, h is the film thickness and ΔU is the sliding speed. For an elliptical contact the corresponding expression is

$$\bar{\eta} = \frac{T}{\pi ab}\frac{h}{\Delta U}, \tag{6b}$$

where $2a$ is now the length of the axis of the ellipse in the rolling direction and $2b$ the length of the axis in the transverse direction. In the present work these are the minor and major axes respectively. If the lubricant is in an elastic condition, a shear modulus, $\bar{G}$, is defined by

$$\bar{G} = ThU/2a^2\Delta U \tag{7a}$$

for line contact, or by

$$\bar{G} = 3ThU/8a^2b\Delta U \tag{7b}$$

for elliptical contact. U is the rolling speed; T is now the total tractive force.

(*b*) *A simplified derivation of the parameters*, α *and* τ_0

When the response in the linear region is elastic, the values of α and τ_0 may be derived from experiments made in the nonlinear region at higher degrees of slip. Thus, at high shear stress, (1) is readily rearranged to give

$$\dot{s} = A\exp\left[(\tau v_\tau - p v_p - E)/k\theta\right], \tag{8}$$

where $\dot{s}$ is the rate of shear. A plot of shear stress against $\ln \dot{s}$ then gives a straight line of slope τ_0. This method of measuring τ_0 has been found to work well at low rolling speeds (Johnson & Tevaarwerk 1977). However, at the speeds used in the present investigation it is not usually satisfactory because the region of isothermal behaviour becomes restricted by the greater viscous heating; τ_0 cannot then be determined accurately. However since the viscosity, η_0, at atmospheric pressure is given by

$$\eta_0 = \tfrac{1}{2}\tau_0 A^{-1}\exp(E/k\theta) \tag{9}$$

(8) can be rearranged to give

$$\tau = (v_p/v_\tau)\,p + \tau_0 \ln(2\eta_0\,\dot{s}/\tau_0), \tag{10}$$

if it is assumed that τ_0 is independent of pressure. Experiment shows that when the shear stress at a fixed rate of shear is plotted against pressure a straight line relation is obtained; this implies that the ratio of the activation volumes v_p/v_τ $(=\alpha\tau_0)$ remains constant over the range of pressure of the experiments. The intercept on the shear stress axis then determines the quantity $\tau_0 \ln(2\eta_0\,\dot{s}/\tau_0)$, and hence τ_0. Since the value of $\alpha\tau_0$ is given by the slope (v_p/v_τ) of the line, a mean value $\bar{\alpha}$ of the pressure coefficient of viscosity may also be derived.

Equation (10) can be rearranged to give an expression for the coefficient of traction, T/W, where W is the load. Thus

$$\frac{T}{W} = \frac{\tau}{p} = \frac{v_p}{v_\tau} + \frac{\tau_0}{p}\ln\frac{2\eta_0\,\dot{s}}{\tau_0}$$

$$= \bar{\alpha}\tau_0 + \frac{\tau_0}{p}\ln\frac{2\eta_0\,\dot{s}}{\tau_0}. \tag{11}$$

It is immediately obvious that as the pressure tends to infinity the traction coefficient tends to a limiting value of $\bar{\alpha}\tau_0$. It has been shown in an earlier paper (Hirst & Moore 1979) that in practical conditions $2\eta_0\dot{s} < \tau_0$ and that it is convenient to rewrite (11) in the form

$$\frac{T}{W} = \bar{\alpha}\tau_0 - \frac{\tau_0}{p}\ln\frac{\tau_0}{2\eta_0\dot{s}}. \tag{12}$$

For a constant value of the product $\bar{\alpha}\tau_0$, the traction coefficient may therefore be expected to fall with an increase of τ_0 or decrease of p.

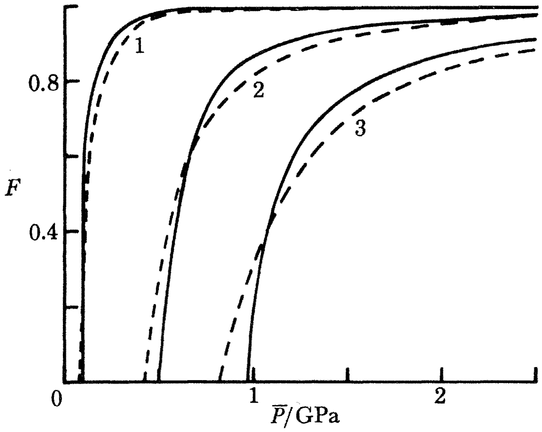

FIGURE 1. Fraction, F, of contact area in highly non-Newtonian régime as a function of mean pressure, $\bar{P}$. ——, Line contact; -----, point contact. $\theta_s = 30$ °C; $\dot{s} = 10^5$ s^{-1}. (1) Santotrac 40, $\bar{\alpha} = 70$ GPa^{-1}; (2) di(2-ethylhexyl)phthalate, $\bar{\alpha} = 15$ GPa^{-1}; (3) di(2-ethylhexyl)sebacate, $\bar{\alpha} = 9.1$ GPa^{-1}.

(c) *The pressure distribution*

It will be assumed in this paper that the pressure distribution is Hertzian. The shear stress then varies over the contact and it follows that when it has to be supposed that $\exp(\tau/\tau_0) \gg \exp(-\tau/\tau_0)$, as in (4), (8) and (10), the condition will not be satisfied over the whole area. To find the proportion of the contact area over which the assumption is valid, let it be supposed to be so when

$$\exp(\tau/\tau_0) > 100\exp(-\tau/\tau_0) \quad \text{or} \quad \tau > 2.303\,\tau_0.$$

Since

$$\sinh(\tau/\tau_0) = (\eta_0\dot{s}/\tau_0)\exp(\alpha p)$$

the condition holds when the pressure exceeds a value P', given by

$$P' = \frac{1}{\alpha}\ln\frac{10\tau_0}{\eta_0\dot{s}}. \tag{13}$$

The fraction F of the contact for which p exceed P' is then given by

$$F = X = \left[1-\left(\frac{P'}{P_0}\right)^2\right]^{\frac{1}{2}} \tag{14}$$

for line contact and by $F = X^2$ for point contact conditions; P_0 is the maximum Hertzian pressure.

For large values of α, and at high pressure, the fraction F approximates to unity and can be assumed to be so, within experimental error. However, when the temperature is raised, the value of α decreases and at high temperatures, especially in experiments in point contact conditions, the value of F may differ quite significantly from unity. In such circumstances, the true values of τ_0 are quite appreciably greater than the apparent values obtained by applying (8) and (10) directly to the experimental results. To illustrate the magnitude of the effect, figure 1 shows the fraction of the contact area over which the condition in (13) is satisfied, or exceeded, for the three fluids Santotrac 40, di(2-ethylhexyl) phthalate and di(2-ethylhexyl)sebacate. The values of α and τ_0 that have been used are taken from line contact measurements made at 30 °C (Hirst & Moore 1979). It will be seen that for the Santotrac fluid, for which $\bar{\alpha} = 70\,\mathrm{Gpa}^{-1}$, the condition is satisfied over almost the whole area and range of pressures whilst with di(2-ethylhexyl)phthalate ($\bar{\alpha} = 15/\mathrm{GPa}^{-1}$) it is effectively satisfied at pressures of greater than about 1 GPa. However for di(2-ethylhexyl)sebacate, for which $\bar{\alpha}$ is only 9.1 GPa^{-1}, the value of F is clearly significantly different from unity over most of the experimental range of pressure. For this last fluid, the theoretical methods outlined earlier are clearly only applicable when the pressure is very high. More appropriate methods of treatment for lower pressures or higher temperatures are described later.

(*d*) *Viscous heating*

The analysis of the traction described so far is based on the assumption of isothermal behaviour. At low degrees of slip this is effectively true and, in general, appreciable shear heating of the lubricant only occurs in the non-linear region when the slip is relatively great. The total rise in temperature, $\Delta\theta_\mathrm{t}$, of the lubricant my be expressed

$$\Delta\theta_\mathrm{t} = \Delta\theta_\mathrm{s} + \Delta\theta_\mathrm{f}, \tag{15}$$

where $\Delta\theta_\mathrm{s}$ is the average temperature rise of the disk surfaces and $\Delta\theta_\mathrm{f}$ the mean rise in temperature of the oil film above that of the disks. The temperature rise, $\Delta\theta_\mathrm{f}$, is given by

$$\Delta\theta_\mathrm{f} = \tau\dot{s}h^2/12K', \tag{16}$$

where K' is the thermal conductivity of the oil (Hirst & Moore 1974). For present purposes, a suitable value of $\Delta\theta_\mathrm{s}$ for elliptical contacts is given by the average temperature rise along the axis of length $2b$. If a uniform rate of generation of heat in the contact is again assumed, this temperature rise is given by

$$\Delta\theta_\mathrm{s} = 0.5\,\tau\dot{s}h(a/k\rho cU)^{\frac{1}{2}}, \tag{17}$$

where K, ρ and C are the thermal conductivity, density and specific heat of the disks, respectively. The same rise in temperature is obtained if a more rigorous calculation is made by assuming a parabolic distribution for the generation of heat along both axes and by averaging over the area.

It will be noted that $\Delta\theta_\mathrm{f}$, which contributes most to $\Delta\theta_\mathrm{t}$ at low disk temperatures, is particularly sensitive to the film thickness; this is because heat is conducted much more rapidly from thin films than from thick ones. At the high rolling speeds used in the greater part of this investigation h is also relatively large and so, therefore, are the rises in temperature. It will be shown later that the effect of these temperature rises on the magnitude of the traction is nevertheless small, particularly when compared with the effect of pressure. It follows that $\bar{\alpha}$ and τ_0 can be derived by application of (10), with little loss of accuracy. In contrast, application of (8) is likely, in most instances, to give seriously misleading results. This is because the rate of increase of shear stress with shear rate is generally very low in the extremely nonlinear region and can be significantly reduced by the effects of shear heating. In a few instances it was possible to apply (8) and so compare values of τ_0 derived by the two different methods.

3. Experimental

(a) *Apparatus*

Two disk machines have been used in this investigation: one being the line contact apparatus described earlier (Hirst & Moore 1978), the other a new point contact machine designed for use at higher speeds and temperatures. The two machines are similar in many respects, consisting essentially of two rollers loaded together in such a way that one of them may deflect elastically in a direction normal to the load. This very small deflection is calibrated and used to define the traction.

In the line contact apparatus the rollers are cylindrical and are made either of tungsten carbide or of steel, hardened to 750 V.P.N. They are 76.2 mm in diameter and are flat to within 0.25 μm across their 19 mm widths. The tungsten carbide disks have a surface finish of 0.005 μm centre line average (c.l.a.) and the steel disks a finish of 0.015 μm. The speed range of this apparatus extends to 11 m/s.

The point contact apparatus made use of the same ancilliary equipment as the line contact machine (see Hirst & Moore 1978). However, since it was primarily intended for work at higher temperatures it was designed to operate over a higher range of speed to counteract the reduction in film thickness which would otherwise accompany the fall in viscosity. This was conveniently done by increasing the diameter of the disks from 76.2 mm to 152.4 mm which also gave more room for fitting the heaters. The heating elements comprised coils of nickel/chrome resistance wire in fibreglass insulation, threaded through a series of 2.4 mm diameter holes parallel to the axis and 12 mm from the circumferential surface of the disks. Two further series of 6.3 mm diameter holes, closer to the disk centres, reduced the rate at which heat was conducted into the drive shaft. Connection of the heating elements to the power supply was made by leads running through the centres of the drive shafts to insulated brass rings and thence through carbon brushes. At the maximum applied voltage of 65 V, about 300 W are dissipated in each disk. This is sufficient to raise the temperature of the disk surfaces to over 150 °C.

In the line contact apparatus the disks were aligned by supporting one of them in a hydrostatic bearing. This design was unsuitable for use at higher temperatures and it was omitted in the new machine. Instead, one disk was crowned and there was then no need for an aligning mechanism. The crowning radius of 229 mm resulted in an elliptical zone of contact having an aspect ratio, b/a, of 3.23. Because of the increased complexity of the disks they were made of steel. In the line contact apparatus the correction for the compliance of the disks was made by the methods already described (Hirst & Moore 1978). For the crowned disks, the compliance was estimated by the method used by Johnson & Roberts (1974) for circular contacts; according to these authors the correction to be applied is given in non-dimensional form by the expression

$$m_{\mathrm{D}}(=TU/W\Delta U) = 1.78\,G_{\mathrm{s}}/P_0, \tag{18}$$

where G_{s} is the shear modulus of the disks and P_0 is the peak Hertz pressure. The method of correction was then the same as described previously with the exception that this theoretical value was used in place of an experimentally determined value.

(b) *The lubricants*

The nine lubricants examined are Santotrac 40, 11,13-dioctyl-11-methyl-tricosane (which will be referred to as the C_{40} hydrocarbon for brevity), dibenzylethylbenzene, di(2-ethylhexyl)-

sebacate and the proprietary experimental fluids MCS 1924, MCS 1925, MCS 1926, MCS 1927 and MCS 1928. The variation of the viscosity, η, with temperature, θ, is described by the Vogel equation

$$\eta = A\exp[B/(\theta+C)] \tag{19}$$

and the variations with pressure, p may be described approximately by the Barus equation

$$\eta = \eta_0 \exp(\alpha p), \tag{20}$$

where η_0 is the viscosity at atmospheric pressure and α the pressure coefficient of viscosity. Values of η_0 and α at 30 °C and of the constants A, B and C are given in table 1. In each case α describes the rise in viscosity between 0 and 0.2 GPa.

Table 1

liquid	η_0/(Pa s)	α/GPa^{-1}	A/(10^{-5} Pa s)	B/°C	C/°C
Santotrac 40	0.0300	34.5	9.94	598.6	74.8
C_{40} hydrocarbon	0.0342	13.6	4.50	1079	133
dibenzylethylbenzene	0.0221	21.1	20.9	462.7	69.3
di(2-ethylhexyl)sebacate	0.0147	13.0	11.1	669.8	107.5
MCS 1924	0.0597	44.2	5.39	742.3	75.9
MCS 1925	0.0109	18.2	8.96	550.2	84.6
MCS 1926	0.0472	31.2	5.17	850.0	94.7
MCS 1927	0.0106	22.7	5.75	746.4	113
MCS 1928	0.0577	24.8	5.44	836.7	90.1

(c) *Experimental conditions*

The methods of measuring the peak traction and the relation between traction and sliding speed have been described in earlier work (Hirst & Moore 1978) together with the capacitance method of measuring the film thickness in the line contact apparatus. Experiments with the line contact machine were all made at a disk surface temperature, θ_s, of 30 °C, and at rolling speeds U ($U = \frac{1}{2}(U_1+U_2)$ where U_1 and U_2 are the individual disk speeds) of up to 10 m/s. The non-linear region was investigated at a rolling speed of 5 m/s unless otherwise stated. Experiments with the point contact machine were generally made at disk temperatures within the range 50–130 °C, but a few were made at 30 °C. Rolling speeds ranged up to 25 m/s, most of the work being conducted at a speed of 10 m/s.

where η_s is the viscosity of the oil at the disk surface temperature, θ_s, R' is the effective radius of the disks ($=\frac{1}{2}$ the actual radius), W the load per unit disk width and E' the effective elastic modulus (equal to $E/(1-\sigma^2)$, where E is the Young modulus and σ the Poisson ratio). As in earlier work (Hirst & Moore 1978) the rate of increase of film thickness with $U\eta_s$ was in good agreement with theory, up to a film thickness of about 1 µm; thereafter it was somewhat smaller. The effect of load on film thickness was also in agreement with theory. Figure 2 shows the effect of load for Santotrac 40 and the C_{40} hydrocarbon; for each, the exponent of W is -0.16. This is identical to the theoretical variation of h_m given by (21) and (22) when the variation of density with pressure is taken to accord with the Dowson & Higginson (1961) expression

$$\frac{\rho_p}{\rho_0} = 1 + \frac{0.581p}{1+1.68p}, \tag{23}$$

where p is the pressure in GPa.

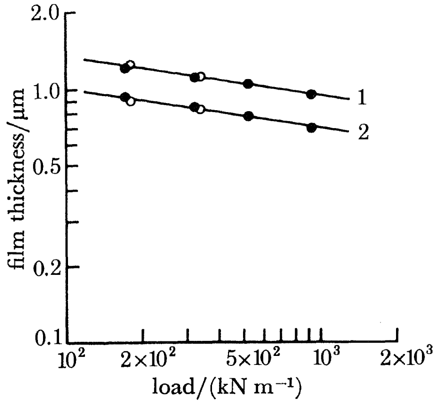

FIGURE 2. Effect of load on film thickness. $U\eta_s = 0.20$ N/m; $\theta_s = 30$ °C. ●, Tungsten carbide disks; ○, steel disks. (1) Santotrac 40; (2) C_{40} hydrocarbon.

In the point contact apparatus the measurement of film thickness at higher temperatures presented difficulty because the capacitances and conductances to be measured were outside the range of the available equipment. However, in view of the results obtained with the line contact apparatus, it was felt that satisfactory estimates would be given by elastohydrodynamic theory. Hamrock & Dowson (1977) give the film thickness, h_c, at the centre of the contact as

$$h_c = 2.69\alpha^{0.53}(U\eta_s)^{0.67}(R'_x)^{0.46}K/W^{0.067}(E')^{0.073}, \tag{24}$$

where R'_x is the effective radius in the direction of rotation, $K = 1-0.61\exp(-0.73\,k)$ and k is the aspect ratio b/a. For the experimental configuration used here $K = 0.942$ and h_c is thus only 5.8 % smaller than the equivalent value for line contact. At the rolling speed of 10 m/s and temperature of 50 °C estimates of h_c ranged from 0.52 to 1.70 µm for the nine fluids while at 110 °C they varied from 0.15 to 0.36 µm.

(*b*) *Traction*

(i) *The validity of the model*

In the line contact experiments with tungsten carbide disks all the fluids except the C_{40} hydrocarbon exhibited elastic behaviour at 30 °C. Values of the modulus, derived by using (7*a*), were then found to vary with the pressure in ways similar to those found in earlier work (Hirst & Moore 1979). It was clear from the point contact experiments, however, that most of the fluids returned to a viscous condition as the temperature was raised, at least when the pressure was low.

The transition is engendered by the rapid decrease in relaxation time, λ, which accompanies the decrease in the Newtonian viscosity, η_N. This is, in turn, due mainly to the reduction in the pressure coefficient of viscosity, α, which is well known from high pressure viscometer studies and is confirmed by the present experiments. The result is that the relaxation time soon becomes smaller then the duration of the experiment.

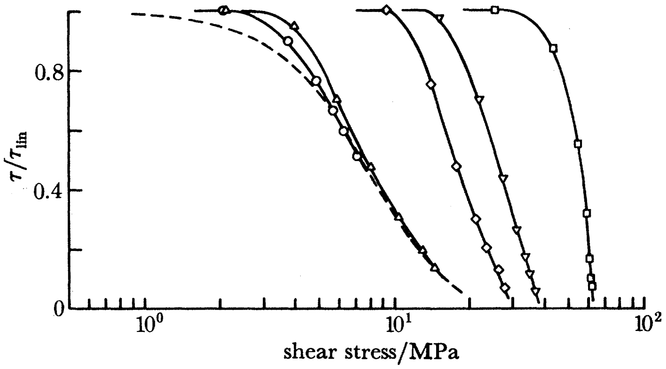

FIGURE 3. Variations in the nonlinear behaviour of MCS 1925 with temperature and pressure. $\theta_s = 110$ °C; $\bar{P}$/GPa: ○, 0.580; △, 0.692; ◇, 0.846; ▽, 0.946. $\theta_s = 50$ °C; $\bar{P}$/GPa: □, 0.946. ----, Eyring curve for $\tau_0 = 3.5$ MPa and constant pressure.

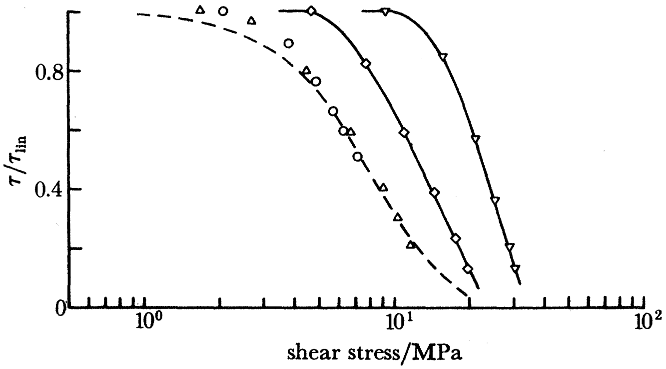

FIGURE 4. Effect of temperature on the nonlinear characteristics of MCS 1925. $\bar{P} = 0.580$ GPa; θ_s/°C: ▽, 50; ◇, 70; △, 90; ○, 110. ----, Eyring curve for $\tau_0 = 3.5$ MPa and constant pressure.

The change from elastic to viscous behaviour in the linear region is also reflected in the nonlinear character of the fluids. This is exemplified by figures 3 and 4, which demonstrate the behaviour of MCS 1925 over a range of temperatures and pressure. The ratio τ/τ_{lin} in these figures quantifies the extent to which the shear stress at a particular shear rate has fallen below the value it would have attained if the relation had remained linear. For purely viscous shear this is equal to the ratio, η/η_N, of the apparent to the Newtonian viscosity and may be expressed in the terms of the Eyring theory as

$$\frac{\eta}{\eta_N} = \frac{\tau/\tau_0}{\sinh(\tau/\tau_0)}. \tag{25}$$

This Eyring relation is illustrated by the dashed lines in figures 3 and 4. Figure 3 shows that at low temperature and high pressure the fluid departs rapidly from linearity as the stress is increased, in a manner which suggests the yielding of a plastic solid. As the temperature rises and pressure falls the behaviour changes progressively towards that of a viscous liquid. At the temperature of

110 °C and pressure of 0.580 GPa the effective viscosity derived by using (6*b*) is only 7.9 Pa s, which is well below the level where significant elastic effects could be expected. The behaviour of the fluids under these conditions is seen to approach quite closely that predicted by (25), except at the lowest shear stresses. The latter, however correspond to values of the traction that lie close to the lower limit of the apparatus, where the experimental error is therefore greater.

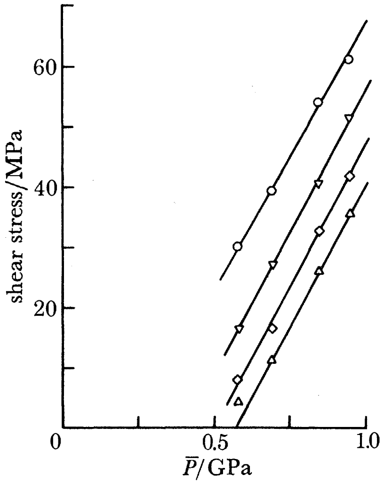

FIGURE 5. Relations between shear stress and pressure for MCS 1925. ○, $\theta_s = 50$ °C, $\dot{s} = 2.13 \times 10^5\ \mathrm{s^{-1}}$; ▽, $\theta_s = 70$ °C, $\dot{s} = 2.00 \times 10^5\ \mathrm{s^{-1}}$; ◇, $\theta_s = 90$ °C, $\dot{s} = 3.00 \times 10^5\ \mathrm{s^{-1}}$; △, $\theta_s = 110$ °C, $\dot{s} = 6.67 \times 10^5\ \mathrm{s^{-1}}$.

(ii) *Complications in the derivation of* α *and* τ_0

In the régime of very nonlinear behaviour, (10) shows that at a constant rate of shear, $\dot{s}$, the shear stress varies linearly with the pressure when $\bar{\alpha}\tau_0 (= v_p/v_r)$ is constant. (Although this is strictly true only when τ_0 is also constant, it can be shown that the inference remains valid, in practice, even when τ_0 varies.) Such linear relations are seen for MCS 1925 at various temperatures in figure 5 and they suggest that the value of $\bar{\alpha}\tau_0$ for the fluid is not substantially affected either by the change in temperature or the pressure. It will be noted, however, that at the highest temperature the point corresponding to the lowest pressure lies above the line drawn through the points for higher pressures. For this condition the fraction F defined by (14) is only 0.23 and a difference in behaviour is therefore not unexpected.

Figure 4 shows that the behaviour of MCS 1925 in these conditions (i.e. at 110 °C and 0.580 GPa) can be fitted fairly closely by a sinh-law curve corresponding to a value of τ_0 of 3.5 MPa. This agrees rather poorly with the value of 6.6 MPa derived by the alternative method of (10), i.e. from the intercept on the shear stress axis of the line for 110 °C in figure 5. However, for a fluid in the viscous condition the viscosity is an exponential function of pressure and the shear force consequently becomes concentrated at the region of highest pressure. It thus follows that the characteristic stress, τ_0, will first be attained at the point of maximum pressure and that thereafter the domain of nonlinear behaviour will gradually spread to the remainder of the contact as the shear rate, $\dot{s}$ is increased. In this circumstance the apparent value of τ_0 obtained by assuming a uniform pressure and shear stress over the whole contact area will be appreciably lower than its true value.

When deriving τ_0 from the experimental relations between shear stress and rate of shear it is thus necessary to take account of the variation in contact pressure. This can be done by assuming a Hertzian distribution, which defines the pressure at each coordinate x, y as

$$p = P_0\left[1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2\right]^{\frac{1}{2}}, \tag{26}$$

and by assuming that the viscosity is defined by (2). The mean contact stress is then

$$\begin{aligned}\bar{\tau} &= \frac{1}{\pi ab}\int_{-b}^{b}\int_{-a}^{a}\tau\,\mathrm{d}x\,\mathrm{d}y\\ &= \frac{1}{\pi ab}\int_{-b}^{b}\int_{-a}^{a}\tau_0\ln\left[\phi+(1+\phi^2)^{\frac{1}{2}}\right]\mathrm{d}x\,\mathrm{d}y,\end{aligned} \tag{27}$$

where

$$\phi = \dot{s}\eta_0\exp{(\alpha p)}/\tau_0$$

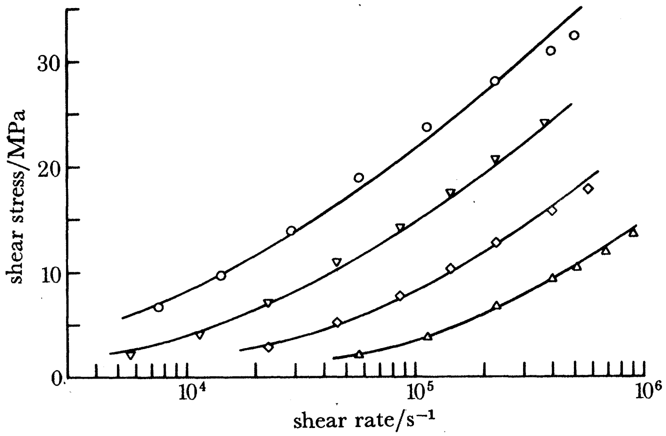

Figure 6. Calculated (——) and experimental relations between shear stress and shear rate for di(2-ethylhexyl)sebacate. θ_s = 50 °C; U = 5.00 m/s; $\bar{P}$/GPa: △, 0.846; ◇, 0.946; ▽, 1.083; ○, 1.227.

and the origin is taken to be the centre of the contact. In the linear region $\bar{\tau}$ is independent of τ_0 and depends only on α; the latter may then be deduced by fitting the calculated shear stress/shear rate relationship for trial values of α to that observed experimentally. In the non-linear region $\bar{\tau}$ depends on both α and τ_0. Once α has been determined, a value for τ_0 can be found that gives the best fit between the calculated and measured non-Newtonian behaviours.

For MCS 1925 the value of τ_0 at 110 °C and 0.580 GPa then becomes 8.0 GPa instead of the former value of 3.5 MPa. It will be shown later that the value of 6.6 MPa derived by the alternative method of (10) also requires upward correction because the fraction F lies well below unity over an appreciable part of the range of pressure. The major disparity between the results obtained by the two methods is thus removed.

(iii) *Low viscosity fluids*

For di(2-ethylhexyl)sebacate and the C_{40} hydrocarbon the pressure coefficients of viscosity are particularly low and the fluids remain in a viscous condition throughout the range of pressure. Values of α and τ_0 could therefore be derived by application of (26) and (27) over an unusually wide range of conditions. Figure 6 illustrates the level of agreement between the calculated and measured shear stresses for di(2-ethylhexyl)sebacate at the temperature of 50 °C and at pressures varying from 0.846 GPa to 1.227 GPa. Figure 7 shows similar plots for the C_{40} hydrocarbon at the pressure of 1.083 GPa and at temperatures in the range 50–110 °C. It is clear that for viscous liquids the Eyring model provides a satisfactory general description of the relation between shear stress and shear rate.

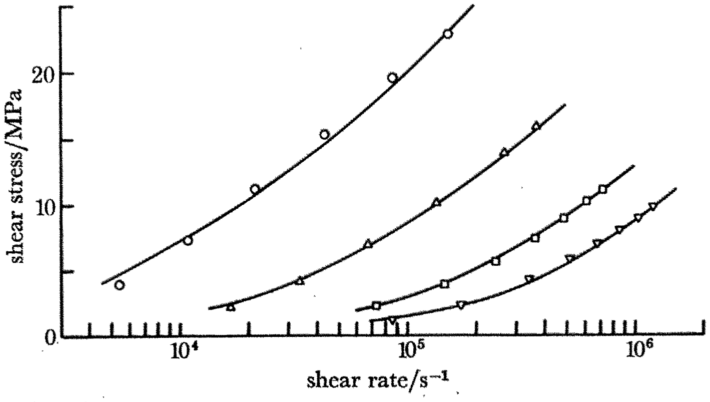

FIGURE 7. Calculated (——) and experimental relations between shear stress and shear for the C_{40} hydrocarbon. $\bar{P}$ = 1.083 GPa; U = 10.0 m/s; θ_s/°C: ○, 50; △, 70; □, 90; ▽, 110.

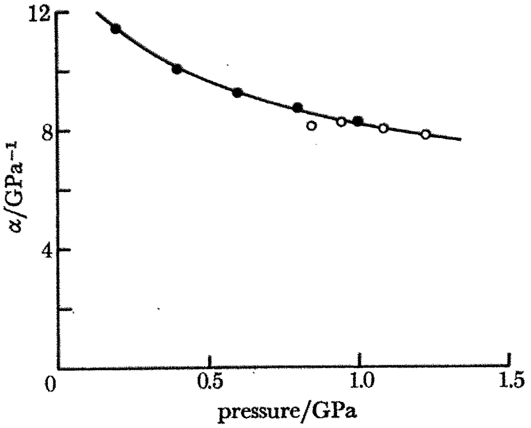

FIGURE 8. Variation in pressure coefficient of viscosity with pressure for di(2-ethylhexyl)sebacate at 50 °C: ●, from A.S.M.E. data; ○, from disk machine data.

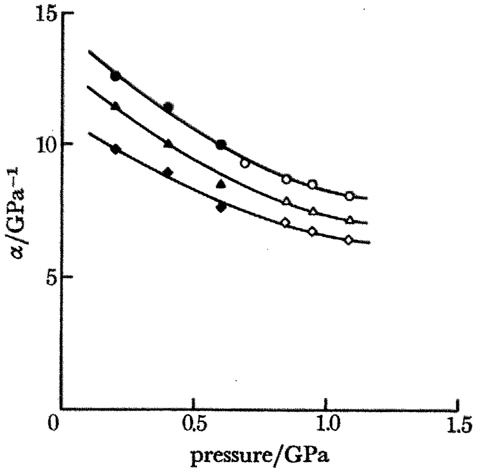

FIGURE 9. Variations in pressure coefficient of viscosity with pressure for the C_{40} hydrocarbon.

temperature	Monsanto data	disk machine data
50 °C	●	○
70 °C	▲	△
90 °C	◆	◇

An interesting feature of di(2-ethylhexyl)sebacate is that its viscosity can also be measured over a very wide range of conditions in a conventional high-pressure viscometer (see A.S.M.E. 1953). The values of α obtained from disk machine experiments can thus be compared directly with values derived from experiments conducted over much longer periods of time. In the latter case, the pressure-viscosity coefficient may be derived by using the expression

$$\alpha_1 = p^{-1} \ln (\eta_p/\eta_0), \tag{28}$$

where η_p is the viscosity at pressure p and η_0 is, again, the value at atmospheric pressure. The comparison, shown in figure 8, indicates agreement to within the error associated with the disk machine data. (It should be noted that the values of α derived from the elastohydrodynamic experiments have been plotted against the mean Hertz pressure. The level of agreement is not greatly affected if the representative pressure (Crook 1963) is used instead.) A similar comparison for the C_{40} hydrocarbon is seen in figure 9. In this case the pressure ranges do not overlap but it is clear that the results from the two types of measurement are mutually consistent.

TABLE 2

lubricant	θ_s/°C	$\bar{P}$/GPa	α/GPa^{-1}	τ_0/MPa
di(2-ethylhexyl)sebacate	50	0.846	8.1	11.8
	50	0.946	8.2	11.8
	50	1.083	8.0	11.8
	50	1.227	7.75	11.8
C_{40} hydrocarbon	50	0.692	9.5	8.0
	50	0.846	8.7	10.0
	50	0.946	8.5	10.0
	50	1.083	8.1	11.5
	70	0.846	7.9	11.0
	70	0.946	7.45	11.0
	70	1.083	7.15	11.0
	90	0.846	7.1	12.0
	90	0.946	6.75	11.5
	90	1.083	6.4	11.5
	110	1.083	6.1	12.0

The values of α and τ_0 obtained for di(2-ethylhexyl)sebacate and the C_{40} hydrocarbon are listed in table 2. It is evident that, at 50 °C, τ_0 for the ester remains constant as the pressure rises whereas for the hydrocarbon it increases. At higher temperatures τ_0 becomes constant for the latter fluid also.

The fluid di(2-ethylhexyl)sebacate was also examined in the point contact apparatus at a temperature of 30 °C. At this condition, however, the behaviour of the fluid could not be analysed in the above way at the higher contact pressures as its low shear behaviour was clearly viscoelastic. (The apparent conflict with earlier line contact results, where pronounced elastic effects were not evident below 1.5 GPa (Hirst & Moore 1979), results from differences in the distribution of pressure. At $\bar{P} = 1.227$ GPa, for example, P_0 is 1.562 GPa for the line contact distribution and 1.841 GPa for point contact. The corresponding viscosities differ by more than an order of magnitude.) In addition the values of 6.3 MPa and 11.5 GPa^{-1} derived, respectively, for τ_0 and $\bar{\alpha}$ by means of (10) were in serious disagreement with the values of 11.1 MPa and 9.1 GPa^{-1} derived from the earlier line contact experiments carried out at high pressure. The discrepancy arises from the fact that, over the pressure range of the point contact experiments (0.846–1.227 GPa) the fraction F is much smaller than unity (see figure 1). Under these conditions, therefore, (2) cannot be simplified to the form of (10). When the mean shear stresses were calculated by using the full expressions for viscosity, (2), and pressure, (26), in conjunction with the values of τ_0 and α given by the earlier line contact measurements, the results were in substantial agreement with experiment over the whole range of pressure (see figure 10). It will also be noted that when F is low the relation between shear stress and pressure ceases to be linear. An effect of this kind has previously been noted for MCS 1925 at 110 °C (see figure 5).

(iv) *Fluid MCS* 1925

It is thus apparent that, even at the low values of F where the relation between shear stress and pressure is no longer linear, it is possible to deduce mean values of τ_0 and $\bar{\alpha}$ which adequately describe the experimental results. Moreover, only one combination of these parameters will be consistent with the experimental results. It was therefore decided to compute for MCS 1925 the relation between shear stress and pressure by using the full expression for viscosity and taking account of the pressure distribution. τ_0 and $\bar{\alpha}$ were treated as disposable constants. The comparison between the experimental data and best fit curves in figure 11 show that the level of agreement obtained is very satisfactory. It is also evident that the complete calculations again correctly predict the departure from linearity occurring at small values of the shear stress. The values of $\bar{\alpha}$ and τ_0 derived by this means are compared in table 3 with the approximate values obtained, by using (10), from the slope and intercept of the shear stress/pressure relations of figure 5. It will be noted that at 50 °C the approximate values are in good agreement with the results obtained by the more elaborate method. As the temperature rises, however, a progressively increasing disparity is evident and at 110 °C the approximate value of τ_0 of 6.6 MPa substantially underestimate the correct value of 9.5 MPa. The latter, however, is not unreasonably different from the value of

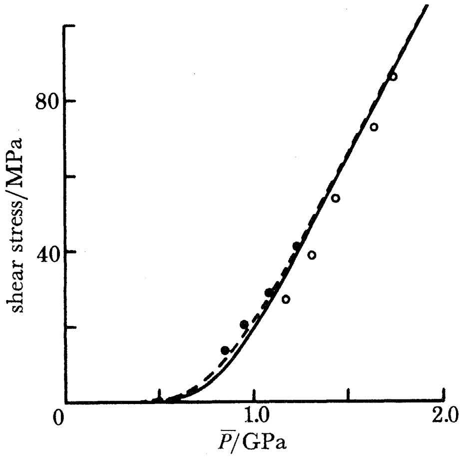

FIGURE 10. Calculated and measured variations in shear stress with pressure for di(2-ethylhexyl)sebacate. θ = 30 °C; $\dot{s} = 10^5\ \text{s}^{-1}$; $\bar{\alpha}$ = 9.1 GPa^{-1}; τ_0 = 11.1 MPa. ------, Point contact calculations; ——, line contact calculations. ●, Point contact measurements; ○, line contact measurements.

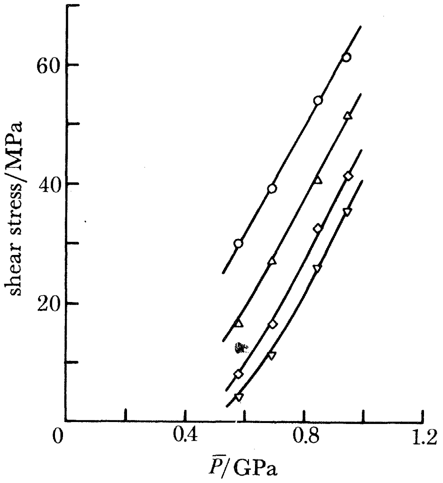

FIGURE 11. Experimental and calculated (——) relations between shear stress and pressure for MCS 1925. ○, θ = 50 °C, $\dot{s} = 2.13 \times 10^5\ \text{s}^{-1}$; △, θ = 70 °C, $\dot{s} = 2.00 \times 10^5\ \text{s}^{-1}$; ◇, θ = 90 °C, $\dot{s} = 3.00 \times 10^5\ \text{s}^{-1}$; ▽, θ = 110 °C, $\dot{s} = 6.67 \times 10^5\ \text{s}^{-1}$.

TABLE 3. $\bar{\alpha}$ AND τ_0 FOR MCS 1925

	θ_s/°C				
	30	50	70	90	110
$\bar{\alpha}$/GPa^{-1} from (10)	35.2⁽¹⁾	29.0	20.2	17.9	14.5
$\bar{\alpha}$/GPa^{-1} from complete calculation	—	27.7	18.8	14.1	12.1
τ_0/MPa from (10)	2.53⁽¹⁾	3.09	4.60	5.26	6.60
τ_0/MPa from complete calculation	—	3.30	5.20	8.00	9.50

(1) Line contact results (steel disks).

8.0 MPa derived, at the lowest pressure of 0.580 GPa, from the variation of the shear stress with the shear rate. Moreover, since the value of τ_0 may be expected to increase with the pressure (see §5) the agreement is probably as good as can be expected.

(v) *Other fluids*

The foregoing calculations have shown plainly that the values of α and τ_0, derived on the assumption of constant shear stress and constant pressure, equal to the mean, can be seriously in error at higher temperature despite the fact that at 30 °C they may be valid. When analysing the results obtained with the remaining fluids, these assumptions were therefore abandoned. Instead the pressure distribution was considered to be Hertzian and the viscosity to obey the sinh relation of (2). The values of α and τ_0 were then derived by finding the best fit to the curves relating the experimental values of the shear stress τ and the mean pressures $\bar{P}$, at a constant rate of shear.

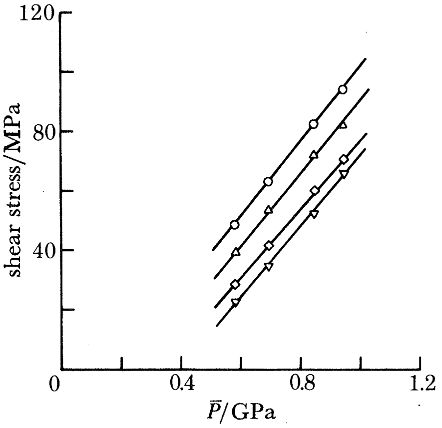

FIGURE 12. Experimental and calculated (——) relations between shear stress and pressure for MCS 1926. ○, $\theta = 50$ °C, $\dot{s} = 3.51 \times 10^5\ \mathrm{s}^{-1}$; △, $\theta = 70$ °C, $\dot{s} = 6.35 \times 10^4\ \mathrm{s}^{-1}$; ◇, $\theta = 90$ °C, $\dot{s} = 1.50 \times 10^5\ \mathrm{s}^{-1}$; ▽, $\theta = 110$ °C, $\dot{s} = 2.22 \times 10^5\ \mathrm{s}^{-1}$.

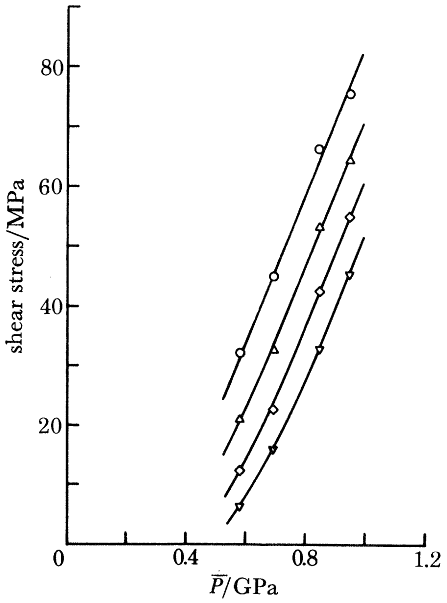

FIGURE 13. Experimental and calculated (——) relations between shear stress and pressure for MCS 1927. ○, $\theta = 50$ °C, $\dot{s} = 1.54 \times 10^5\ \mathrm{s}^{-1}$; △, $\theta = 70$ °C, $\dot{s} = 2.35 \times 10^5\ \mathrm{s}^{-1}$; ◇, $\theta = 90$ °C, $\dot{s} = 3.33 \times 10^5\ \mathrm{s}^{-1}$; ▽, $\theta = 110$ °C, $\dot{s} = 4.21 \times 10^5\ \mathrm{s}^{-1}$.

Figures 12 and 13 for the fluids MCS 1926 and MCS 1927 demonstrate the excellence of the fit which is obtained. The values of $\bar{\alpha}$ and τ_0 obtained by this method of calculation for the fluids dibenzylethylbenzene (d.b.e.b.), Santotrac 40, MCS 1928, MCS 1927, MCS 1926, MCS 1925 and MCS 1924 are listed in table 4. This table also shows, for interest and comparison, the values given by the approximate method. In some instances the accuracy of the latter is good enough to render correction unnecessary; this particularly applies to the line contact results obtained at 30 °C. It should be noted, however, that in these cases the values of $\bar{\alpha}$ are relatively large, the smallest being 26.8 GPa^{-1}. At the higher temperatures the approximate values of τ_0 are up to 40 % too small and correction is clearly essential.

At 110 °C and at low pressure the behaviour of most of the fluids is viscous in the linear régime and it is possible to derive a value of α from the variation of shear stress with shear rate.

(MCS 1924 is exceptional; as table 4 shows, it has a particularly high pressure coefficient of viscosity and consequently retains a viscoelastic character.) The values of α' obtained by this method are compared in table 5 with those obtained at the same temperature for $\bar{\alpha}$, both sets of values having been derived by the complete method of calculation. Also shown are the values, α_1, estimated by means of equation (28) from conventional viscometric experiments at low

TABLE 4

	θ_s/°C	$\bar{\alpha}$/GPa^{-1}	τ_0/MPa	$\bar{\alpha}$/GPa^{-1}	τ_0/MPa
lubricant		approximate		full calculation	
dibenzylethylbenzene	30	76.7(1)	0.86(1)	—	—
	50	44.6	1.48	43.0	1.50
	70	26.3	2.51	25.4	2.60
	90	17.8	4.04	16.5	4.55
	110	15.2	4.74	13.4	6.05
	130	13.3	5.92	11.5	8.00
Santotrac 40	30	67.2(1)	1.83(1)	—	—
	50	51.5	2.41	49.6	2.55
	70	35.5	3.52	33.4	3.80
	90	26.9	4.76	24.4	5.50
	110	20.5	6.45	18.2	7.75
MCS 1928	30	37.5(1)	2.56(1)	—	—
	50	26.6	3.65	23.9	4.10
	70	21.9	4.30	18.4	5.50
	90	18.4	4.94	15.0	7.00
	110	16.2	5.26	13.0	8.00
MSC 1927	30	26.8(1)	4.29(1)	—	—
	50	34.8	4.95	22.5	5.75
	70	20.3	5.85	17.7	7.50
	90	17.5	6.70	14.8	9.50
	110	15.5	7.46	13.1	11.0
MSC 1926	30	50.6(1)	2.63(1)	—	—
	50	40.5	3.01	38.9	3.30
	70	32.1	3.74	29.7	4.20
	90	24.3	4.77	22.5	5.30
	110	20.7	5.64	18.7	6.90
MCS 1924	30	96.4(1)	1.26	—	—
	50	90.0	1.21	—	—
	70	72.3	1.41	—	—
	90	47.4	2.13	43.5	2.43
	110	30.5	3.51	27.0	4.10

(1) Line contact results (steel disks).

pressures. The quantities α_1, α' and $\bar{\alpha}$ each define a mean value of the pressure coefficient, typically corresponding to pressure ranges of 0–0.2 GPa, 0–0.6 GPa and 0–0.8 GPa, respectively. It will be noted that for each fluid there is a consistent tendency for α either to increase or to decrease as the pressure range increases. Di(2-ethylhexyl)sebacate and the C_{40} hydrocarbon have been seen in figures 8 and 9, to behave in a similar manner at lower temperatures. Table 6 shows that at 30 °C more marked changes in α occur as the pressure range increases.

TABLE 6. PRESSURE COEFFICIENTS OF VISCOSITY AT 30 °C

lubricant	$\alpha_1/\mathrm{GPa}^{-1}$	$\bar{\alpha}/\mathrm{GPa}^{-1}$
Santotrac 40	34.5	67.2
dibenzylethylbenzene	21.1	76.7
C_{40} hydrocarbon	13.6	11.6
MCS 1924	44.2	96.4
MCS 1925	18.2	35.2
MCS 1926	31.2	50.6
MCS 1927	22.7	26.8
MCS 1928	24.8	37.5

5. DISCUSSION

(a) *General*

The results for each of the nine fluids show that both the characteristic shear stress, τ_0, and the pressure coefficient of viscosity, α, vary with the temperature. For di(2-ethylhexyl)sebacate and the C_{40} hydrocarbon α is also seen to vary with the pressure, whereas τ_0 generally remains constant.

One of the notable features of this investigation is how greatly the physical condition of the lubricant changes when both pressure and temperature are varied. For the range of conditions represented in figure 5, for example, the equilibrium viscosity of MCS 1925 varies from 56 Pa s to 6×10^{14} Pa s at the point of maximum pressure. While it is unlikely that the latter value is actually attained in practice, its magnitude does convey the glassy nature of the fluid under the relevant conditions and shows why it behaves more as a solid than as a liquid. No method of deriving α and τ_0 has been found that is applicable over the whole range of conditions. Thus at low temperatures, where α and therefore η_{N} are generally high, the non-Newtonian viscous response of the fluid is masked at low rates of shear by its elastic behaviour. The nonlinear viscous properties become evident only when η/η_{N} is very low and the condition $\exp(\tau/\tau_0) \gg \exp(-\tau/\tau_0)$ obtains. In these circumstances (8) and (10) are applicable. It should be noted that when (10) applies the mean shear stress, which is measured experimentally, is nearly proportional to the mean contact pressure. The application of (10) is therefore straightforward. At high temperatures, however, α tends to be low and the behaviour of the fluid becomes entirely viscous. The non-linear behaviour is then described by (2) and neither (8) nor (10) holds. Isothermal conditions for which $\exp(\tau/\tau_0) \gg \exp(-\tau/\tau_0)$ can no longer be found because of the intrusive effects of shear heating at the high rates of shear which would be necessary. The interpretation of experiment is further complicated by the approximately exponential variation of shear stress with pressure over the zone of contact. No simple relation between the measured shear stress and the mean Hertz pressure then exists and the properties of the lubricant can only be deduced by means of more elaborate calculations, based on (26) and (27).

Nevertheless, despite these difficulties, the values of α and τ_0 obtained by the different methods are mutually consistent. Moreover the pressure coefficient of viscosity determined for di(2-ethylhexyl)sebacate and the C_{40} hydrocarbon agree satisfactorily with those obtained from conventional viscometric experiments.

(b) *Thermal effects*

The values of α and τ_0 that have been obtained have been derived from experiments made in conditions chosen to be approximately isothermal. In general the magnitude of the traction must be affected to some extent by the effects of shear heating within the oil film and it is necessary to determine how important these effects may be. The authors have inferred in earlier work that the peak traction, $T_{\max}$, is determined primarily by the intrinsic non-Newtonian properties of the lubricant and only to a secondary extent by the effects of shear heating (Hirst & Moore 1979). This suggests that, on the ascending part of the traction curve, at least, the thermal effects may be treated quite simply.

TABLE 7

$\overline{P}$/GPa	$10^{-4}\dot{s}/\mathrm{s}^{-1}$	$\theta_s/°\mathrm{C}$	$\Delta\theta_t/°\mathrm{C}$	$\bar{\tau}(\theta_s)$/MPa	$\bar{\tau}(\bar{\theta})$/MPa
0.580	5	50	2.4	52.6	51.4
0.946	5	50	4.9	98.1	95.6
0.580	20	110	0.5	21.2	20.8
0.946	20	110	1.7	68.8	67.5

If, then, the oil within the elastohydrodynamic contact is considered to experience the average temperature rise, $\Delta\theta_t$, defined by (15), its mean temperature, $\bar{\theta}$, rises to $\theta_s+\Delta\theta_t$. By taking account of the dependence of $\bar{\theta}$ upon the shear rate and on α, τ_0 and η_0, (27) may be extended to define the relation between shear stress and shear rate in the thermal régime. This merely requires the substitution in (27) of $\tau_0(\theta)$ in place of τ_0 and redefining ϕ as $\phi = \dot{s}\eta_0(\theta)\exp[\alpha(\theta)p]/\tau_0(\theta)$. For Santotrac 40, for example, the results in this paper may be represented by the relations

$$\tau_0 = 1.06\times10^6\exp(0.0181\theta) \tag{29}$$

and

$$\bar{\alpha}\tau_0 = 0.125+5.02\times10^{-6}(\theta-50)^2, \tag{30}$$

where τ_0 is in Pa and θ in °C. $\eta_0(\theta)$ is defined by (19). A simple iterative cycle based on (15), (27) and (26) rapidly converges to give the mean shear stress for the specified contact conditions.

To ascertain, first, the reliability of the experimental methods used to derive the lubricant properties, the influence of shear heating in the nominally isothermal region was assessed. Table 7 gives the magnitudes of the temperature rises, $\Delta\theta_t$, for the range of conditions at which τ_0 and $\bar{\alpha}\tau_0$ were derived. In addition it compares the calculated shear stresses, $\bar{\tau}(\bar{\theta})$, at the temperatures $\theta_s+\Delta\theta_t$ with the corresponding isothermal values, $\bar{\tau}(\theta_s)$. It is clear that although these temperature rises cannot be dismissed as negligible, as was generally the case in earlier work at room temperature, their influence on the magnitude of the shear stress nevertheless remains small. At 50 °C they lead to a reduction in the mean shear stress of 2.3 % at the lowest pressure and 2.5 % at the highest. At 110 °C the corresponding reductions are 1.9 % at both pressures. Such disturbances are no greater than the experimental error and their neglect does not therefore have any significant effect on the derived values of τ_0 and $\bar{\alpha}$.

Calculations have also been made for higher rates of shear in the region of the peak value of the traction. Values of the peak mean shear stress, $\tau_{\max}$, given by these calculations, at temperatures ranging from 30 °C to 110 °C and pressures of 0.490 GPa to 0.946 GPa, are compared with experiment in figure 14. Although the calculated values do not agree precisely with the relations observed experimentally the general level of agreement is nevertheless very good. Similarly, the modest reduction in $\tau_{\max}$ with increasing rolling speed seen experimentally is also predicted fairly well (see figure 15).

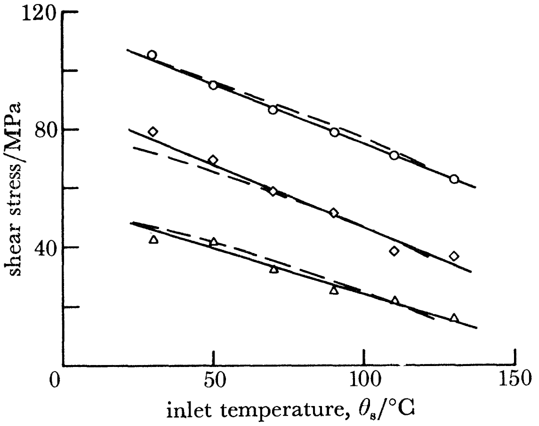

FIGURE 14. Calculated (––––) and measured variations in peak shear stress with disk surface temperature for Santotrac 40. $U = 10$ m/s; $\overline{P}$/GPa: △, 0.480; ◇, 0.692; ○, 0.94.

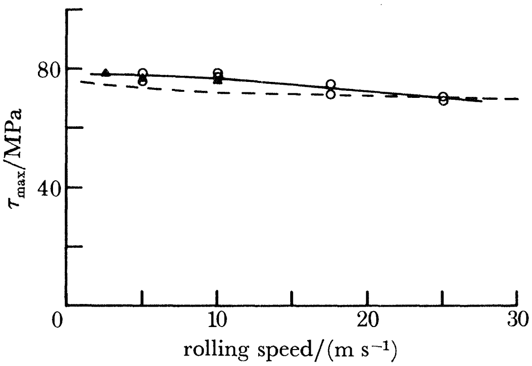

FIGURE 15. Calculated (––––) and observed dependences of peak shear stress on rolling speed for Santotrac 40. $\overline{P} = 0.692$ GPa; $\theta_s = 30$ °C; ○, Elliptical contact measurements; ▲, line contact measurements.

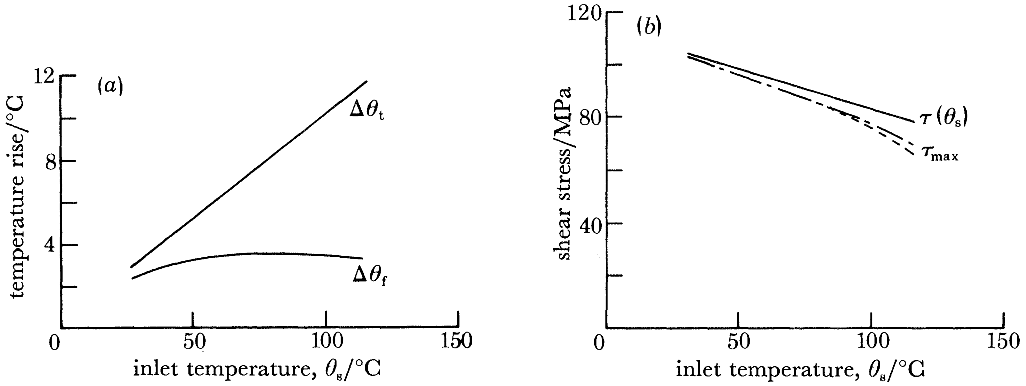

FIGURE 16. (*a*) Calculated temperature rises at the point of maximum traction. $\overline{P} = 0.946$ GPa; $U = 10$ m/s; fluid: Santotrac. (*b*) Comparison between peak shear stresses and isothermal stresses at same shear rate. $\overline{P} = 0.946$ GPa, $U = 10$ m/s. ———, Complete calculations; – – –, approximate calculations.

A noteworthy feature of these calculations is that the temperature rises corresponding to the maximum values of the traction are not very large. Figure 16*a* shows that at 0.946 GPa pressure they range from about 3 °C at a disk temperature of 30 °C to about 11 °C at a disk temperature of 110 °C. It thus follows, and is confirmed in figure 16*b*, that the maximum shear stress sustained by the lubricant is not very different from the isothermal value defined by its intrinsic non-Newtonian properties. It should also be noted that where there are discrepancies between the calculated and the experimental values, the imperfection is more likely to be in the rheological description of the fluid than in the method of calculation of the rise in temperature. The linear experimental relation depicted in figure 14, for example, appears to be peculiar to high traction fluids.

Finally, it is of interest to note that a good first approximation to the tractive capacity of a fluid can often be obtained by using the general form of (10), i.e.

$$\bar{\tau} = \bar{\alpha}\tau_0(\theta)\,\bar{P} - \tau_0(\theta)\ln[\tau_0(\theta)/2\eta_0(\theta)\,\bar{s}]. \tag{31}$$

Estimates of $\tau_{\max}$ for Santotrac 40 obtained by means of (31) together with (15), (29), (30) and (19) are shown for the pressure 0.946 GPa in figure 16*b*. It will be noted that below the disk temperature of 90 °C no significant differences are seen between these results and those obtained by the more elaborate method.

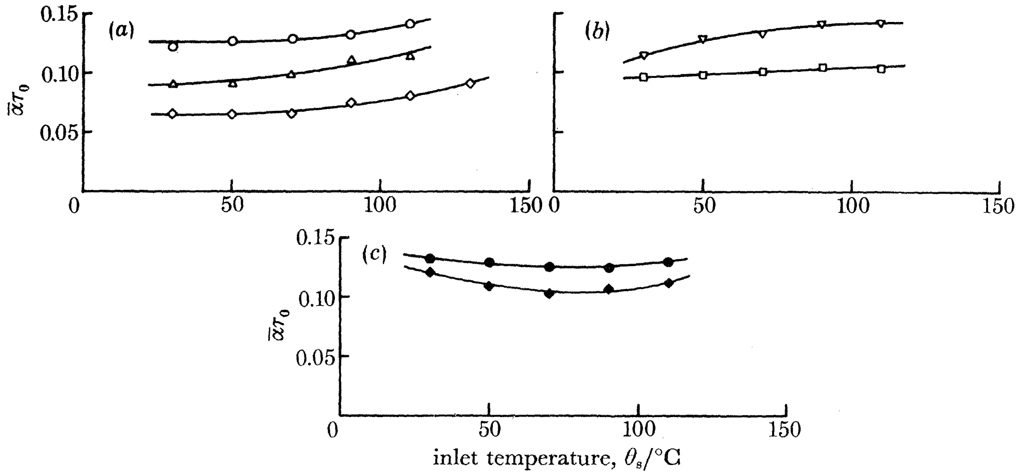

FIGURE 17. Effect of temperature on the limiting traction coefficient, $\bar{\alpha}\tau_0$ ($= v_p/v_\tau$). (*a*) ○, Santotrac 40; △, MCS 1925; ◇, dibenzylethylbenzene. (*b*) ▽, MCS 1927; □, MCS 1928. (*c*) ●, MCS 1926; ◆, MCS 1924.

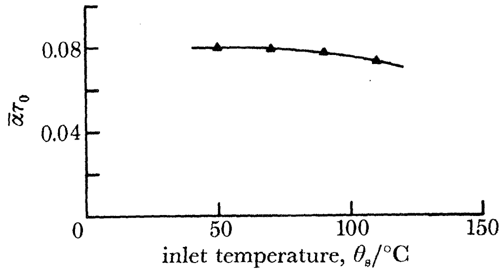

FIGURE 18. Effect of temperature on the limiting traction coefficient for the C_{40} hydrocarbon.

(*c*) *The traction coefficient*

Since the effect of shear heating on the magnitude of the maximum traction is quite small, the traction depends predominantly on the values of α, τ_0 and p while the variation of the traction with temperature depends on the variation of α and τ_0 with temperature. Figures 11, 12 and 13 show the relations between the shear stress, τ, and the mean pressure, $\bar{P}$, at constant rate of shear for several temperatures. According to (10) the slope is given by (v_p/v_τ) which, by definition, is equal to the product $\alpha\tau_0$. The figures show that the slopes are almost independent of temperature and this leads immediately to the result that the limiting value of the traction coefficient at high pressure should be independent of temperature (see (12)). (This result may appear surprising in view of the very great changes in the viscosity of the fluid over the range of temperature.) Since

$\alpha\tau_0$ remains the same while α decreases with temperature, it is clear that τ_0 must increase sufficiently to compensate. Equation (12) then predicts that the effect of an increase in temperature is not to alter the limiting value of the traction coefficient but to raise the pressure at which it can be attained. It would be interesting to extend the experimental work to higher pressures to find whether this prediction can be verified.

Although this argument has been presented in its simplest form, on the basis of the applicability of (10) and (12), more sophisticated calculations taking into account the pressure distribution do not change the conclusions. Thus, figure 17 shows the variation of $\alpha\tau_0$ with temperature for the values of α and τ_0 derived from the experimental data of figures 11–13. It also includes values of $\alpha\tau$ derived in a similar way for the fluids Santotrac 40, dibenzylethylbenzene, MCS 1924 and MCS 1928. The behaviour of the C_{40} hydrocarbon is shown in figure 18. It will be noticed that the values of $\alpha\tau_0$ do not change greatly over the range of temperature. If anything there is a slight tendency for the values to increase with temperature but there is no consistent tendency exhibited by all the fluids.

In general, therefore, the broad conclusion arising out of this work is that the limiting value of the traction coefficient at high pressure is effectively independent of the temperature. The changes in the behaviour of the fluids at lower pressures then result primarily from the changes in the individual parameters α and τ_0. These vary quite considerably with temperature and, of the two, it is particularly rewarding to consider the variation of the characteristic stress τ_0.

TABLE 8. ACTIVATION VOLUMES FOR THE C_{40} HYDROCARBON

$\theta_s/°\mathrm{C}$	$v_\tau/10^{-28}\ \mathrm{m}^3$	$0.5\ v_\mathrm{m}/10^{-28}\ \mathrm{m}^3$
50	4.3	4.8
70	4.3	4.8
90	4.3	4.8
110	4.4	4.9

(*d*) *The variation of τ_0 with temperature*

Inspection of (1) and (2) shows that there is a correspondence between τ_0 and an activation volume $v_\tau (= kT/\tau_0)$ which, according to the Eyring theory, is equal to half the molecular volume, v_m. The fluid di(2-ethylhexyl)sebacate is of particular interest. In earlier work (Hirst & Moore 1979) a mean value $\tau_0 = 11.1$ MPa was obtained at 30 °C giving a corresponding activation volume, v_τ, of $3.8 \times 10^{-28}\ \mathrm{m}^3$. In the present work a slightly higher value of 11.8 MPa was obtained at 50 °C, equivalent to the same activation volume of $3.8 \times 10^{-28}\ \mathrm{m}^3$. At both temperatures τ_0 and hence v_τ are independent of the pressure (see table 2 for example). At 30 °C and 50 °C, $\frac{1}{2}v_\mathrm{m}$ for di(2-ethylhexyl)sebacate is $3.9 \times 10^{-28}\ \mathrm{m}^3$ and $4.0 \times 10^{-28}\ \mathrm{m}^3$ respectively, at atmospheric pressure and about $3.1 \times 10^{-28}\ \mathrm{m}^3$ and $3.2 \times 10^{-28}\ \mathrm{m}^3$ respectively, at the high pressures of the measurements in the disk machine. (Here, as elsewhere, the Dowson & Higginson expression, (23), is used to make approximate corrections for the effect of pressure on density). Thus, for simple liquids, v_τ is predicted to depend only weakly on the temperature, so weakly in fact as to be within the present experimental error. For this fluid, then, the Eyring theory accounts satisfactorily for the observed non-Newtonian behaviour.

For the C_{40} hydrocarbon the experimental results are similar in kind to those for di(2-ethylhexyl)sebacate. Apart from a slight variation at 50 °C, τ_0 was again found to be independent of the pressure. The corresponding activation volumes, v_τ, are listed in table 8 and are again seen to be independent of temperature, to within the experimental accuracy. Good agreement is again seen between the measured values of v_τ and the values predicted by the Eyring theory. It will be noted that when v_τ is constant, τ_0 is directly proportional to the absolute temperature.

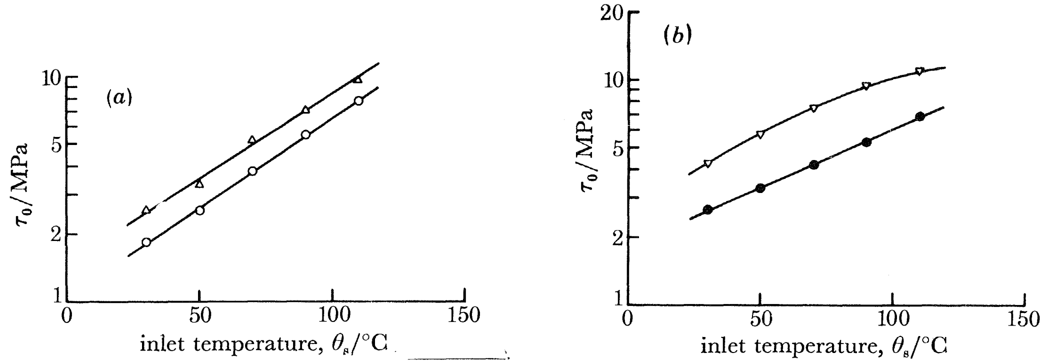

FIGURE 19. Effect of temperature on τ_0. (a) △, MCS 1925; ○, Santotrac 40; (b) ▽, MCS 1927; ●, MCS 1926.

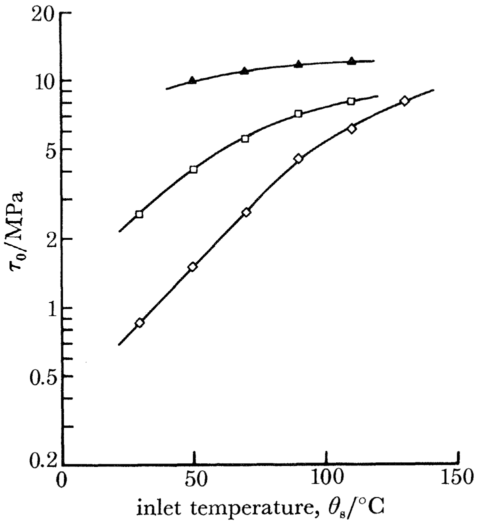

FIGURE 20. Effect of temperature on τ_0. △, C_{40} hydrocarbon; □, MCS 1928; ◇, dibenzylethylbenzene.

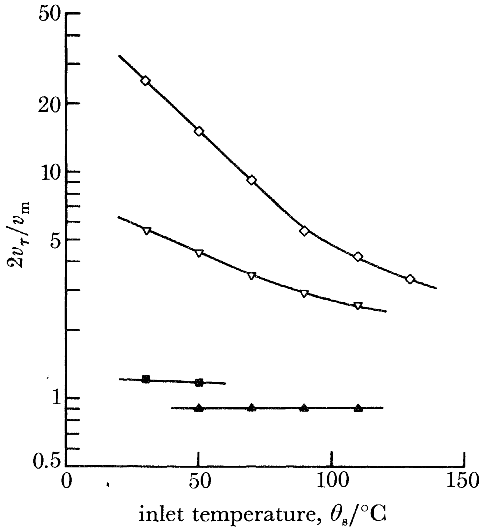

FIGURE 21. Effect of temperature on the shear activation volume, v_τ, in relation to $\frac{1}{2}v_m$. ◇, dibenzylethylbenzene; ▽, MCS 1927; ■, di(2-ethylhexyl)sebacate; ▲, C_{40} hydrocarbon.

For the remaining seven fluids the behaviour of τ_0 is quite different. The average values given in figure 19 show that for MCS 1925, Santotrac 40 and MCS 1926 the relation with temperature is exponential. Rather similar behaviour is seen for MCS 1927 but in this case the rate of increase of τ_0 slows down as the temperature rises. Figure 20 compares the behaviour of the C_{40} hydrocarbon with that of MCS 1928 and dibenzylethylbenzene. As has already been stated, τ_0 for the C_{40} hydrocarbon is proportional to the absolute temperature. Dibenzylethylbenzene exhibits an exponential increase in τ_0 with temperature at lower temperatures but at higher temperatures τ_0 increases less rapidly. The behaviour of MCS 1928 appears to be intermediate in nature between that of dibenzylethylbenzene and the C_{40} hydrocarbon. In figure 21 values of the activation volume, v_τ, derived from the magnitudes of τ_0 for dibenzylethylbenzene, MCS 1927, di(2-ethylhexyl)sebacate and the C_{40} hydrocarbon are compared with values of $\frac{1}{2}v_m$ estimated from the molecular weights and densities of the fluids. For the latter two fluids the ratio $2v_\tau/v_\mathrm{m}$ lies close to unity and is independent of the temperature, as noted above. For the former two fluids, however, behaviour of a quite different type is seen. At low temperatures $2v_\tau/v_\mathrm{m}$ is appreciably higher than unity and decreases exponentially as the temperature rises. At the higher end of the temperature range, where v_τ is still higher than $\frac{1}{2}v_\mathrm{m}$ but now less markedly so, the rate of decrease becomes lower.

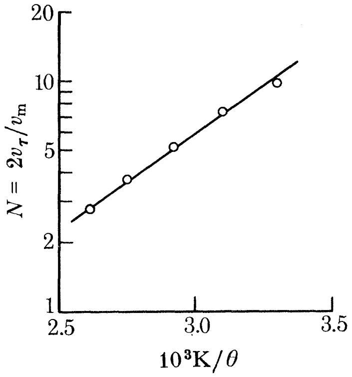

FIGURE 22. Effect of temperature on cluster size for Santotrac 40.

(*e*) *Molecular clustering*

In a previous paper (Hirst & Moore 1979) the authors observed that, whereas some types of fluid gave activation volumes fairly close to $\frac{1}{2}v_\mathrm{m}$, others gave substantially higher values. It was suggested that the latter observation might have resulted from the existence of some form of molecular aggregation. The present results may also be explicable on the same basis. If, to borrow the terminology of Ubbelohde (1965), it is assumed that molecules may associate in a series of step-wise equilibria to form non-crystallizable clusters of various sizes, the average size of the clusters will appear as twice the activation volume, v_τ. To give an indication of the number of molecules in the average molecular cluster, v_τ is better expressed as its ratio, N, to $\frac{1}{2}v_\mathrm{m}$. N is then a measure of the state of equilibrium between the associated and unassociated molecular species and, as such, may be expected to relate to the energy change, ΔH, of the clustering process and the absolute temperature. It is not obvious that the form of the relation should be a simple one but it is interesting to note (see figure 22) that a plot of lg N against $1/\theta$ for Santotrac 40 yields a straight line relationship. This is, of course, the form of temperature dependence expected for simple equilibrium constants.

It therefore seems reasonable to suppose that the exponential dependence of τ_0 on temperature seen in figures 19 and 20 arises from the dissociation of molecular clusters. As the process of dissociation nears completion the rate of change of τ_0 with temperature gradually diminishes until the behaviour of a simple Eyring fluid is attained.

It will be shown in a later paper that the effect of pressure on τ_0 can be explained on the same basis. For unassociated liquids no significant variation in τ_0 is expected when pressure is applied and none is observed. For associated species, however, (i.e. when $N > 1$) N and hence τ_0 vary exponentially. If, as seems likely, ΔH contains a PV term, the equilibrium constant for each association step will also vary exponentially with pressure. Given the similarity between the effect of temperature on N and the behaviour of such equilibrium constants, the similarity in the effects of pressure is not surprising.

The postulate of non-crystallizable clusters in liquids is not new and has been used by

Ubbelohde (1965), for example, to explain the 'excess' viscosity of some liquids near their freezing points. It may be significant that the type of molecule prone to exhibit 'excess' viscosity, i.e. one having a relatively rigid structure and re-entrant features in its shape, is similar to the type that gives high values of N (see Hirst & Moore 1979). The possibility thus exists that the two observations are different manifestations of the same phenomenon.

The magnitude of the activation volume v_p is much more difficult to predict than is v_τ, even for 'simple' liquids which conform to the Eyring theory. It is however worth noting that according to one version of the theory (Kauzmann & Eyring 1940) the activation volume for pressure, v_p, represents the volume which has to be created against the action of the applied pressure to enable a molecule to move to a new position. It seems reasonable to suppose that this volume would be proportional to the molecular size or, if the unit of flow is a molecular cluster, to the size of the molecular cluster. If this assumption is made the constancy of v_p/v_τ and therefore of $\alpha\tau_0$ then follows. Thus, from the molecular viewpoint, the approximate constancy of $\alpha\tau_0$ does not appear to be very unexpected even though, when viewed against the very great changes in the viscosities that can occur when the temperature is changed, at first sight it might seem to be so.

(*f*) *A simpler model of the fluid*

Finally it should be noted that, since the viscosity falls so rapidly with temperature and since the working temperature of machinery is so frequently higher than ambient, it should seldom be necessary in practice to consider the viscoelastic and other quasi-solid properties which lubricants may possess. These are only exhibited at low temperature and high pressure when combined with extremely small rates of shear. For most practical purposes, it is sufficient to treat the fluid simply as a non-Newtonian liquid and the Eyring model then gives a valuable guide to the behaviour to be expected.

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